A Transformer of One-Third Wavelength in Two Sections—For a Frequency and Its First Harmonic

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***Abstract*—The quarter-wave transformer transforms in one frequency f_0, but not its first harmonic $2f_0$. The transform in $2f_0$ is needed in a dual-band operation of GSM and PCS. This requirement can be fulfilled in a two line-sections of $1/6$ wavelengths each, adding to $1/3$ *wavelengths for the total transformer length* in the lower band centering at f_0 (fundamental), or $2/3$ wavelengths in the upper band centering at $2f_0$ (1st harmonic). The $1/3$-wave transformer is analytically inexact but *effectively exact* for engineering applications. For example, for impedance transforms of $K = 4$, the inexactness gives a reflection of $|\Gamma| = 0.013$.**

***Index Terms*—Dual frequencies, one-third wave transformer, transmission-line.**

I. Introduction

WITH advent of the dual-band operation, such as the GSM and PCS bands for mobile telephones, it becomes interesting to have antenna arrays and RF circuits that operate at frequency f_0 and the exactly doubled frequency $2f_0$. The feed system of a dual-band antenna array needs dual-band power dividers with impedance transformers having the same transforming property at f_0 and $2f_0$. A quarter-wave transformer [1] can transform the load resistance R_L to an input resistance KR_L at a design frequency f_0. It can give the same transform for $3f_0$ and all odd harmonics. It cannot, however, give the transform for $2f_0$ and all even harmonics. This letter shows that using two one-sixth-wave sections of transmission line at f_0, the new transformer gives the desired transform at both f_0, $2f_0$, plus all harmonics that are not divisible by three. The characteristic impedance of the quarter-wave transformer is $K^{1/2}R_L$. The characteristic impedances of the 2 one-sixth-wave sections transformer are similar; they are $K^{1/3}R_L$ and $K^{2/3}R_L$.

II. Derivation

The input impedance two-section transmission line in Fig. 1 is given by

$$Z_{in} = R + jX = Z_1 \frac{Z_L' + jZ_1 \tan\beta l_1}{Z_1 + jZ_L' \tan\beta l_1} \tag{1}$$

with

$$Z_L' = Z_2 \frac{R_L + jZ_2 \tan\beta l_2}{Z_2 + jR_L \tan\beta l_2}. \tag{1a}$$

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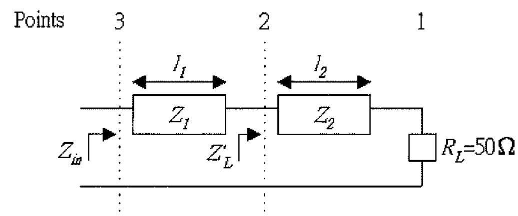

Fig. 1. Circuit of a two-section transmission line forming a transformer.

The complex impedance Z_{in} has two parts, the real R and the imaginary X. Therefore, (1) is actually two equations. The dual frequency has two different β, i.e., β_0 and $2\beta_0$ for f_0, and $2f_0$, assuming lines with no frequency dispersion. Hence, (1) becomes a total of four equations for the four unknowns of Z_1 and l_1, Z_2 and l_2.

The input resistance is taken to be $R = K Z_0$ where K is an arbitrary real constant similar to what one would choose for the regular quarter-wave transformer. So, we can take X to be zero. The four equations of (1) are transcendental. However, they can be solved easily by nonlinear software.

III. Solution

For solution of (1) we choose as the first example $R_L = 50\,\Omega$, $KR_L = 100\,\Omega$ or $K = 2$. The line is assumed to be a coaxial line with no frequency dispersions. The nonlinear solution is that $Z_1 = 79.37\,\Omega$, $l_1 = 0.1667\lambda_0$, $Z_2 = 63.00\,\Omega$ and $l_2 = 0.1667\lambda_0$, where λ_0 is the wavelength at the fundamental f_0. The solution indicates that we should have the equations

$$\begin{aligned} Z_1 &= K^{2/3}R_L, \qquad l_1 = \lambda_0/6, \\ Z_2 &= K^{1/3}R_L \quad \text{and} \quad l_2 = \lambda_0/6. \end{aligned} \tag{2}$$

It shall be shown in the next section that the design equations above are analytically not exact, but they are numerically *nearly exact*.

Continuing on the example, a Smith chart locus, normalized to the 50 Ω of the load R_L, is used to follow the impedance of the example of $KR_L = 100\,\Omega$. That is, the impedance along the transmission line sections in Fig. 1 from the load to the input Z_{in}. In Fig. 2, the Smith chart is plotted for the fundamental frequency of f_0. It is observed that there is a kink in to locus at the junction between the second and the first section of the transmission line before the locus reaches the $SWR = 2$ location of the input, at $KR_L \cong 100\,\Omega$.

In Fig. 3, the Smith chart is plotted for the first harmonic of $2f_0$. It is observed that the locus is twice as long and there is a different kink in the locus at the junction between the second and

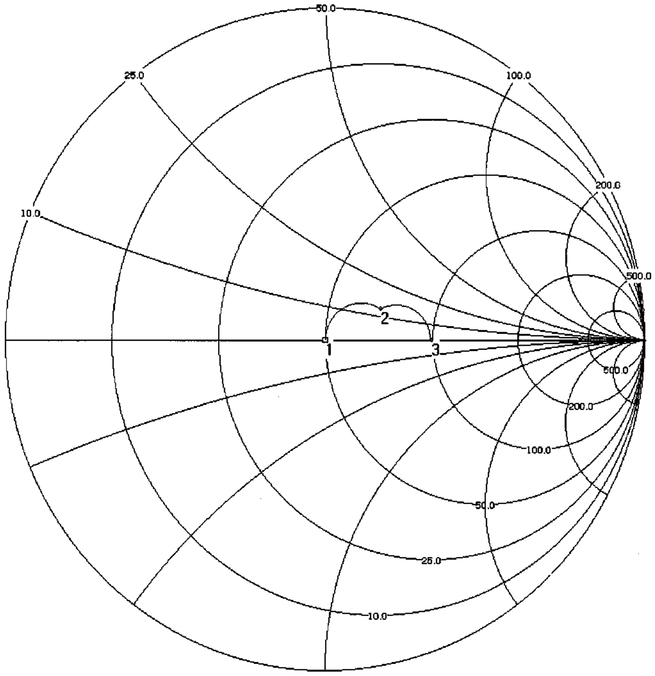

Fig. 2. Impedance locus of the 2-section $1/3$-wave transformer on the Smith chart of $Z_0 = 50\ \Omega$ at the fundamental frequency, f_0. Points 1, 2, and 3 correspond to that in Fig. 1.

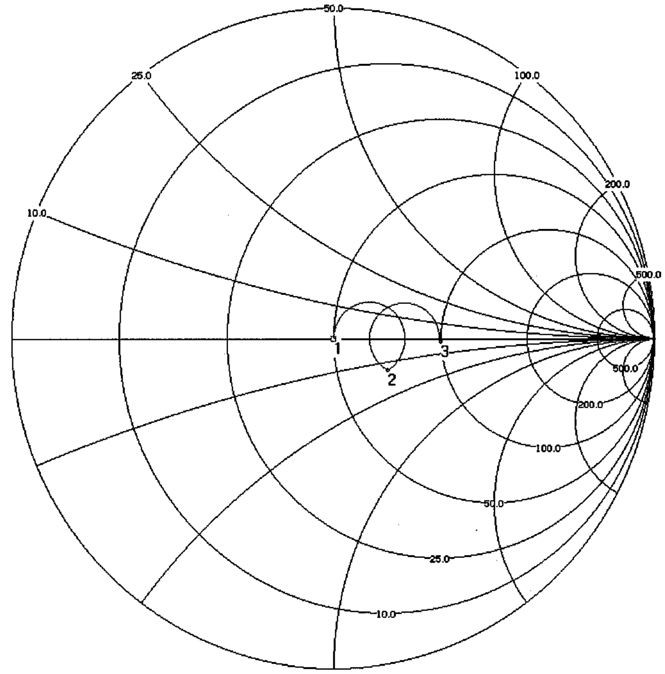

Fig. 3. Impedance locus of the 2-section $1/3$-transformer on the Smith chart of $Z_0 = 50\ \Omega$ at the 1st harmonic frequency, i.e., $2f_0$.

the first section of the transmission line. Still, the locus reaches the $SWR = 2$ location of input $KR_L \cong 100\ \Omega$.

Figs. 2 and 3 have continuous locus sections of $1/3$ or $2/3$ of a circle in the Smith chart for f_0 and $2f_0$. At $3f_0$, each locus section becomes a full circle and as a result there is no transformer action as it is equivalent to the zero frequency. This cyclic property means that the $4f_0$ and $5f_0$ should again have transformer action. In other words, the transformer action occurs at nf_0 where n is an integer not divisible by three.

IV. Analytical Proof of the Design Equations in (2)

Analytically, through applying the software Mathmatica [2] to (1) and (1a), the equations of (2) are tested and are proven to be not exact. Numerically, however, through a series of further examples, it is found that the equations of (2) are indeed valid (i.e., nearly exact) for most designs within stringent tolerances. The results of the examples are given below.

Let K_a be the complex value obtained after (2) with a specified K, is substituted into (1) and (1a). Let $|\Gamma|$ be the reflection coefficient from the deviation of K_a from K. Then, at $K = 2$, $K_a = 2.003 + j0.0054$, $|\Gamma| = 0.0016$; $K = 3$, $K_a = 3.0176 + j0.0332$, $|\Gamma| = 0.0062$; $K = 4$, $K_a = 4.0456 + j0.0911$, $|\Gamma| = 0.01266$; up to $K = 10$, $K_a = 10.392 + j1.1763$, $|\Gamma| = 0.0607$. The above are for the fundamental frequency f_0. For the harmonic $2f_0$, the real part of each K remains unchanged but the imaginary part just changes sign but remains also unchanged in magnitude.

V. Conclusion

Equivalent to the quarter-wave transformer for one frequency, a $1/3$-wave transformer of two sections is found for the frequency f_0 and its first harmonic $2f_0$. The design is obtained numerically through solving a set of 4 nonlinear simultaneous transmission line equations.

Unlike the quarter-wave transformer, the $1/3$-wave transformer can be shown by an analytical software, such as Mathematica [2], that it is not exact. Numerically, on the other hand, the $1/3$-wave transformer is shown in this paper to be "effectively exact" for engineering applications of impedance transform ratios, say, up to $K = 6$ (with a reflection $|\Gamma| = 0.028$) and up to $K = 15$ (with $|\Gamma| = 0.102$). This may be why the $1/3$-wave transformer has not been discovered before from an exact transmission line analysis.

References

[1] D. K. Cheng, *Field and Wave Electromagnetics*. New York: Addison-Wesley, 1989, pp. 497–509.

[2] "Mathematica," Wolfram Research, Inc., version 4.